

# Declarative Multi-paradigm Programming

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Do not no code algorithms and stepwise execution

Describe logical relationships

~> powerful abstractions

- domain specific languages

~> higher programming level

~> reliable and maintainable programs

- pointer structures  $\Rightarrow$  algebraic data types
- complex procedures  $\Rightarrow$  comprehensible parts  
(pattern matching, local definitions)



Declarative languages based on **different formalisms**, e.g.,

## Functional Languages

- lambda calculus
- functions
- directed equations
- reduction of expressions

## Logic Languages

- predicate logic
- predicates
- definite clauses
- goal solving by resolution

## Constraint Languages

- constraint structures
- constraints
- specific constraint solvers



## Functional Languages

- higher-order functions
- expressive type systems
- demand-driven evaluation
- optimality, modularity

## Logic Languages

- compute with partial information
- non-deterministic search
- unification

## Constraint Languages

- specific domains
- efficient constraint solving

All features are useful  $\rightsquigarrow$  declarative multi-paradigm languages



Goal: combine best of declarative paradigms in a single model

- **efficient execution** principles of functional languages (determinism, laziness)
- **flexibility** of logic languages (computation with partial information, built-in search)
- **application-domains** of constraint languages (constraint solvers for specific domains)
- avoid non-declarative features of Prolog (arithmetic, cut, I/O, side-effects)



## Extend logic languages

- add functional notation as syntactic sugar (Ciao-Prolog, Mercury, HAL, Oz, . . .)
- equational definitions, nested functional expressions
- translation into logic kernel
- don't exploit functional information for execution

## Extend functional languages

- add logic features (logic variables, non-determinism) (Escher, TOY, Curry, . . .)
- functional syntax, logic programming use
- retain efficient (demand-driven) evaluation whenever possible
- additional mechanism for logic-oriented computations



As a language for concrete examples, we use

## Curry [POPL'97,...]

- multi-paradigm declarative language
- extension of Haskell (non-strict functional language)
- developed by an international initiative
- provide a standard for functional logic languages (research, teaching, application)
- several implementations and various tools available

↪ <http://www.curry-language.org>



**Functional program:** set of functions defined by equations/rules

```
double x = x + x
```

**Functional computation:** replace subterms by equal subterms

```
double (1+2) ⇒ (1+2) + (1+2) ⇒ 3 + (1+2) ⇒ 3+3 ⇒ 6
```

Another computation:

```
double (1+2) ⇒ (1+2) + (1+2) ⇒ (1+2) + 3 ⇒ 3+3 ⇒ 6
```

And another computation:

```
double (1+2) ⇒ double 3 ⇒ 3+3 ⇒ 6
```





```
double x = x + x
```

$$\begin{aligned} \underline{\text{double } (1+2)} &\Rightarrow \underline{(1+2)} + (1+2) \Rightarrow 3 + \underline{(1+2)} \Rightarrow \underline{3+3} \Rightarrow 6 \\ \underline{\text{double } (1+2)} &\Rightarrow (1+2) + \underline{(1+2)} \Rightarrow \underline{(1+2)} + 3 \Rightarrow \underline{3+3} \Rightarrow 6 \\ \underline{\text{double } (1+2)} &\Rightarrow \underline{\text{double } 3} \Rightarrow \underline{3+3} \Rightarrow 6 \end{aligned}$$

All derivations  $\rightsquigarrow$  same result: **referential transparency**

- computed result independent of evaluation order
- no side effects
- simplifies reasoning and maintenance

Several strategies: **what are good strategies?**



## Values in declarative languages: terms

```
data Bool = True | False
```

## Definition by pattern matching:

```
not True = False  
not False = True
```

Replacing equals by equals still valid:

```
not (not False) ⇒ not True ⇒ False
```



## List of elements of type $a$

```
data List a = [] | a : List a
```

Some notation:  $[a] \approx \text{List } a$

$$[e_1, e_2, \dots, e_n] \approx e_1 : e_2 : \dots : e_n : []$$

## List concatenation “++”

$$(++)\ :: [a] \rightarrow [a] \rightarrow [a]$$
$$[] \quad ++ \text{ ys} = \text{ys}$$
$$(x : \text{xs}) ++ \text{ys} = x : \text{xs} ++ \text{ys}$$
$$[1, 2, 3] ++ [4] \Rightarrow^* [1, 2, 3, 4]$$



## List concatenation “++”

$$\begin{aligned} (++) &:: [a] \rightarrow [a] \rightarrow [a] \\ [] & \quad ++ \text{ ys } = \text{ ys} \\ (x:\text{xs}) & ++ \text{ ys } = x : \text{xs} ++ \text{ ys} \end{aligned}$$

Use “++” to specify other list functions:

Last element of a list:  $\text{last } \text{xs} = e$  iff  $\exists \text{ys} : \text{ys} ++ [e] = \text{xs}$

Direct implementation in a **functional logic language**:

- search for solutions w.r.t. existentially quantified variables
- solve equations over nested functional expressions

## Definition of `last` in Curry

$$\begin{aligned} \text{last } \text{xs} & \mid \text{ys} ++ [e] =: \text{xs} \\ & = e \quad \text{where } \text{ys}, e \text{ free} \end{aligned}$$



Set of functions defined by **equations** (or **rules**)

$$f\ t_1 \dots t_n \mid c = r$$

$f$  : function name

$t_1 \dots t_n$  : data terms (constructors, variables)

$c$  : condition (optional)

$r$  : expression

Constructor-based term rewriting system

Non-~~constructor~~ based rules  
Rules with extra variables

$$\begin{aligned} \text{last } xs \mid ys++[e] ::= xs \\ = e \quad \text{where } ys, e \text{ free} \end{aligned}$$

Non-constructive, forbidden to provide efficient evaluation strategy  
allowed in contrast to traditional rewrite systems



Rewriting not sufficient in the presence of logic variables  $\rightsquigarrow$

**Narrowing** = variable instantiation + rewriting

**Narrowing step:**  $t \rightsquigarrow_{p, l \rightarrow r, \sigma} t'$

$p$  : non-variable position in  $t$

$l \rightarrow r$  : program rule (variant)

$\sigma$  : unifier for  $t|_p$  and  $l$

$t'$  :  $\sigma(t[r]_p)$

Why not most general unifiers?



## Narrowing with mgu's is not optimal

```
data Nat = Z | S Nat
add Z     y = y
add (S x) y = S (add x y)
leq Z     _ = True
leq (S _) Z = False
leq (S x) (S y) = leq x y
```

$\text{leq } v \text{ (add } w \text{ Z)} \text{ leq } v \text{ (add } w \text{ Z)} \rightsquigarrow_{\{v \mapsto Z\}} \text{True}$

Another narrowing computation:

$\text{leq } v \text{ (add } w \text{ Z)} \rightsquigarrow_{\{w \mapsto Z\}} \text{leq } v \text{ Z leq } v \text{ Z} \rightsquigarrow_{\{v \mapsto S z\}} \text{False}$

And another narrowing computation:

$\text{leq } v \text{ (add } w \text{ Z)} \rightsquigarrow_{\{w \mapsto Z\}} \text{leq } v \text{ Z} \rightsquigarrow_{\{v \mapsto Z\}} \text{True}$  **superfluous!**

Avoid last derivation by **non-mgu** in first step:

$\text{leq } v \text{ (add } w \text{ Z)} \rightsquigarrow_{\{v \mapsto S z, w \mapsto Z\}} \text{leq } (S z) \text{ Z}$



- constructive method to compute positions and unifiers
- defined on **inductively sequential** rewrite systems:  
there is always a discriminating argument
- formal definition: organize rules in *definitional trees* [Antoy'92]
- here: transform rules into `case` expressions

```
add Z      y = y           add x y = case x of
add (S x) y = S(add x y)  =>   Z      → y
                               S z → S(add z y)
```

```
leq Z      _ = True       => leq x y = case x of
leq (S _) Z = False      =>   Z      → True
leq (S x) (S y) = leq x y  S a → case y of
                               Z      → False
                               S b → leq a b
```





## case expressions

- standard compile-time transformation to implement pattern matching
- guide *lazy* evaluation strategy

```
leq x y = case x of Z    → True
                S a → case y of Z    → False
                S b → leq a b
```

## Evaluate function call `leq t1 t2`

- 1 Evaluate  $t_1$  to head normal form  $h_1$
- 2 If  $h_1 = Z$ : return `True`
- 3 If  $h_1 = (S \dots)$ : evaluate  $t_2$  to head normal form
- 4 If  $h_1$  variable: bind  $h_1$  to  $Z$  or  $(S \_)$  and proceed

```
leq v (add w Z)  $\rightsquigarrow_{\{v \mapsto S a, w \mapsto Z\}}$  leq (S a) Z
```



Needed narrowing solves equations  $t_1 =:= t_2$

Interpretation of “ $=:=$ ”:

- **strict equality** on terms
- $t_1 =:= t_2$  satisfied if both sides reducible to same value (finite data term)
- undefined on infinite terms

```
f = 0 : f
g = 0 : g
```

$\rightsquigarrow f =:= g$  does not hold

- constructive form of equality (definable by standard rewrite rules)
- used in current functional and logic languages



Sound and complete (w.r.t. strict equality)

Optimal strategy:

- 1 **No unnecessary steps:**  
Each step is needed, i.e., unavoidable to compute a solution.
- 2 **Shortest derivations:**  
If common subterms are shared, derivations have minimal length.
- 3 **Minimal set of computed solutions:**  
Solutions computed by two distinct derivations are independent.
- 4 **Determinism:**  
No non-deterministic step during evaluation of ground expressions  
( $\approx$  functional programming)

Note: similar results unknown for purely logic programming!



## Non-deterministic choice

$x \text{ ? } y = x$

$x \text{ ? } y = y$

- $0 \text{ ? } 1$  (don't know) evaluates to 0 or 1
- `case` expressions not sufficient (no discriminating argument)
- *weakly needed narrowing* = needed narrowing + choice

## Non-deterministic operations/functions

- interpretation: mapping from values into sets of values
- declarative semantics [González-Moreno et al., JLP'99]
- supported in modern functional logic languages
- advantage compared to predicates: demand-driven evaluation



## Non-deterministic list insertion

```
insert e []      = [e]
insert e (x:xs) = (e : x : xs) ? (x : insert e xs)
```

## Permutations of a list

```
permute []      = []
permute (x:xs) = insert x (permute xs)
```

## Permutation sort

```
sorted []          = []
sorted [x]         = [x]
sorted (x1:x2:xs) | x1 ≤ x2 = x1 : sorted (x2:xs)
psort xs = sorted (permute xs)
```

Reduced search space due to demand-driven evaluation of `(permute xs)`



Advantages of non-deterministic operations as generators:

- demand-driven generation of solutions
- modular program structure, no floundering

```
psort [5, 4, 3, 2, 1]  ~>  sorted (permute [5, 4, 3, 2, 1])
                       ~>*  sorted (5 : 4 : permute [3, 2, 1])
                               _____
                               undefined: discard this alternative
```

**Effect:** Permutations of  $[3, 2, 1]$  are not enumerated!

Permutation sort for  $[n, n-1, \dots, 2, 1]$ : #or-branches/disjunctions

Length of the list:	4	5	6	8	10
generate-and-test	24	120	720	40320	3628800
test-of-generate	19	59	180	1637	14758



Subtle aspect of non-deterministic operations: treatment of arguments

```
coin = 0 ? 1
```

```
double x = x+x
```

```
double coin
```

```
↪ coin+coin    ↪*    0 | 1 | 1 | 2    need-time choice
```

```
↪ double 0 | double 1    ↪*    0 | 2    call-time choice
```

## Call-time choice

- semantics with “least astonishment”
- declarative foundation: CRWL calculus [González-Moreno et al., JLP'99]
- implementation: demand-driven + sharing
- used in current functional logic languages



## Narrowing

- resolution extended to functional logic programming
- sound, complete
- efficient (optimal) by exploiting functional information

Alternative principle:

## Residuation (Escher, Life, NUE-Prolog, Oz, ...)

- evaluate functions only deterministically
- suspend function calls if necessary
- encode non-determinism in predicates or disjunctions
- concurrency primitive required:  
“ $c_1$  &  $c_2$ ” evaluates constraints  $c_1$  and  $c_2$  concurrently





```
add Z      y = y                nat Z      = success
add (S x)  y = S(add x y)      nat (S x) = nat x
```

Evaluate function `add` by residuation:

```
add y Z ::= S Z & nat y nat y
→{y ↦ S x} add (S x) Z ::= S Z & nat x
→{} S (add x Z) ::= S Z & nat x
→{} add x Z ::= Z & nat x
→{x ↦ Z} add Z Z ::= Z & success
→{} Z ::= Z & success
→{} success & success
→{} success
```



## Narrowing

- sound and complete
- possible non-deterministic evaluation of functions
- optimal for particular classes of programs

## Residuation

- incomplete (floundering)
- deterministic evaluation of functions
- supports concurrency (declarative concurrency)
- method to connect external functions

No clear winner  $\rightsquigarrow$  **combine narrowing + residuation**

Possible by adding *flexible/rigid tags* in `case` expressions

- `flexible case`: instantiate free argument variable (narrowing)
- `rigid case`: suspend on free argument variable (residuation)



Narrowing not applicable (no explicit defining rules available)

Appropriate model: residuation

Declarative interpretation: defined by infinite set of rules

## External arithmetic operations

$$0 + 0 = 0$$

$$0 * 0 = 0$$

$$0 + 1 = 1$$

$$1 * 1 = 1$$

$$1 + 1 = 2$$

$$2 * 2 = 4$$

$$\vdots$$
$$\vdots$$

Implemented in some other language:

- rules not accessible
- can't deal with unevaluated/free arguments
- reduce arguments to ground values before the call
- suspend in case of free variable (**residuation**)



Important technique for generic programming and code reuse

## Map a function on all list elements

```
map :: (a -> b) -> [a] -> [b]
map _ []      = []
map f (x:xs) = f x : map f xs

map double [1,2,3] ~>* [2,4,6]
map (\x -> x*x) [2,3,4] ~>* [4,9,16]
```

Implementation:

- primitive operation `apply`:  $\text{apply } f \ e \rightsquigarrow f \ e$
- sufficient to support higher-order functional programming

Problem: application of unknown functions?

- instantiate function variable: costly
- pragmatic solution: function application is **rigid** (i.e., no guessing)



- occur in conditions of conditional rules
- restrict applicability: solve constraints before applying rule
- no syntactic extension necessary:  
constraint  $\approx$  expression of type `Success`

## Basic constraints

```
-- strict equality
```

```
(==) :: a → a → Success
```

```
-- concurrent conjunction
```

```
(&) :: Success → Success → Success
```

```
-- always satisfied
```

```
success :: Success
```

```
last xs | ys++[e]==xs = e where ys,e free
```



Constraints are ordinary expressions  $\rightsquigarrow$  pass as arguments or results

## Constraint combinator

```
allValid :: [Success] → Success
allValid [] = success
allValid (c:cs) = c & allValid cs
```

**Constraint programming:** add constraints to deal with specific domains

## Finite domain constraints

```
domain      :: [Int] → Int → Int → Success
allDifferent :: [Int] → Success
labeling    :: [LabelingOption] → [Int] → Success
```

Integration of constraint programming as in CLP

Combined with lazy higher-order programming



SuDoku puzzle:  $9 \times 9$  matrix of digits

Representation: matrix  $m$  (list of lists of FD variables)

9			2			5		
	4			6			3	
		3						6
			9			2		
				5			8	
		7			4			3
7							1	
	5			2				4
		1			6			9

## SuDoku Solver with FD Constraints

```
sudoku :: [[Int]] → Success
sudoku m = domain (concat m) 1 9
          & allValid (map allDifferent m)
          & allValid (map allDifferent (transpose m))
          & allValid (map allDifferent (squaresOfNine m))
          & labeling [FirstFailConstrained] (concat m)
```



Requirement on programs: constructor-based rules

## Last element of a list

```
last (xs++[e]) = e      -- not allowed
```

Eliminate non-constructor pattern with extra-variables:

```
last xs | ys++[e]==xs = e  where ys,e free
```

Disadvantage: strict equality evaluates *all* arguments

```
last [failed,3]  ~>* failure (instead of 3)
```

Solution: allow **functional patterns** (patterns with defined functions)

Possible due to functional logic kernel!





Functional pattern  $\approx$  set of patterns where functions are evaluated

## Evaluations of $xs++[e]$

$$\begin{aligned} xs++[e] &\rightsquigarrow^*_{xs \mapsto [e]} [e] \\ xs++[e] &\rightsquigarrow^*_{xs \mapsto [x1]} [x1, e] \\ xs++[e] &\rightsquigarrow^*_{xs \mapsto [x1, x2]} [x1, x2, e] \\ \dots \end{aligned}$$

## Interpretation of $last (xs++[e]) = e$

$$\begin{aligned} last [e] &= e \\ last [x1, e] &= e \\ last [x1, x2, e] &= e \\ \dots \end{aligned}$$

- $last [failed, 3] \rightsquigarrow^* 3$
- implementation: demand-driven functional pattern unification
- powerful concept to express transformation problems



```
<contacts>
  <entry>
    <name>Hanus</name>
    <first>Michael</first>
    <phone>0431/8807271</phone>
    <email>mh@informatik.uni-kiel.de</email>
    <email>hanus@acm.org</email>
  </entry>
  <entry>
    <name>Smith</name>
    <first>William</first>
    <nickname>Bill</nickname>
    <phone>+1-987-742-9388</phone>
  </entry>
</contacts>
```

- processing: matching, querying, transformation
- basically term structures, declarative languages seem appropriate
- problems: structure often incompletely specified, evolves over time
- specialized languages: XPath, XQuery, XSLT, Xcerpt [Bry et al. '02]



## XML documents are term structures:

```
data XmlExp = XText String
            | XElem String [(String,String)] [XmlExp]
```

## Useful abstractions

```
xml t c = XElem t [] c
txt s   = XText s
```

```
xml "entry" [xml "name" [txt "Hanus"],
             xml "first" [txt "Michael"],
             xml "phone" [txt "0431/8807271"]]
```

pretty printing ⇒

```
<entry>
  <name>Hanus</name>
  <first>Michael</first>
  <phone>0431/8807271</phone>
</entry>
```



## Extract name and phone number by pattern matching:

```
getNamePhone
  (xml "entry"
    [xml "name" [xtxt name],
     _/
     xml "phone" [xtxt phone]]) = name++": "++phone
```

## Functional patterns improves readability, but still problematic:

- exact XML structure must be known
- many details of large structures often irrelevant
- change in structure  $\rightsquigarrow$  update all patterns

Better: **define appropriate abstractions and use them in functional patterns**



- do not enumerate all children of a structure
- provide flexibility for future structure extensions

```
getNamePhone
(xml "entry"
  (with [xml "name" [xtxt name],
        xml "phone" [xtxt phone]])) = name++": "++phone
```

```
with :: [a] → [a]  -- return some list containing elements
with []      = _
with (x:xs) = _ ++ x : with xs
```

Example: `with [1,2] ~> x1:...:xm:1:y1:...:yn:2:zS`



- order of children unspecified
- provide flexibility for future structural changes

```
getNamePhone
(xml "entry"
  (with
    (anyorder [xml "phone" [xtxt phone],
              xml "name"  [xtxt name]]))) = name++": "++phone
```

```
-- Return a permutation of the input list:
anyorder :: [a] → [a]
anyorder [] = []
anyorder (xs++[x]++ys) = x : anyorder (xs++ys)
```



## Deep pattern

- structure at the root or at a descendant (at arbitrary depth) of the root
- ease queries in complex structures
- provide flexibility for future structural changes

```
getNamePhone
  (deepXml "entry"
    (with [xml "name" [xtxt name],
          xml "phone" [xtxt phone]])) = name++": "++phone
```

```
deepXml :: String → [XmlExp] → XmlExp
deepXml tag elems = xml tag elems
deepXml tag elems = xml _ (_ ++ [deepXml tag elems] ++ _)
```

# Example: XML Pattern Matching at Arbitrary Depth



```
getPhone (deepXml "phone" [xtxt num]) = num
```

```
getPhone (<contacts>
  <entry>
    <name>Hanus</name>
    <first>Michael</first>
    <phone>0431/8807271</phone>
    <email>mh@informatik.uni-kiel.de</email>
    <email>hanus@acm.org</email>
  </entry>
  <entry>
    <name>Smith</name>
    <first>William</first>
    <nickname>Bill</nickname>
    <phone>+1-987-742-9388</phone>
  </entry>
</contacts>)
```

```
↪ "0431-8807271"
```

```
↪ "+1-987-742-9388"
```





## Transformation of Documents

- transform XML documents into other XML or HTML documents
- transformation task almost trivial in pattern-based languages, e.g.:  
*transform pattern = newdoc*

```
transPhone (deepXml "entry" (with [xml "name" [xtxt n],  
                                   xml "first" [xtxt f],  
                                   xml "phone" phone])) =  
  xml "phonename" [xml "phone" phone,  
                  xml "fullname" [xtxt (f++' ':n)]]
```

## Accumulate Results

- accumulation of global or intermediate results
- requires “findall” (*encapsulated search*)



## Encapsulating non-deterministic search is important

- accumulate intermediate results
- select optimal/best solutions
- non-deterministic search between I/O must be encapsulated
- **complication:** demand-driven evaluation + sharing + “findall”

```
let y=0?1 in findall (...y...)
```

- evaluate “0?1” inside or outside the capsule?
- order of solutions might depend on evaluation time

Declarative capsule: **set functions**



## Idea

Associate to any operation  $f$  a new operation  $f_S$  (**set function**)

- $f_S$  computes set of all values computed by  $f$
- $(f_S e) \approx$  sets of all non-deterministic values of  $(f v)$  if  $v$  is a value of  $e$
- capture non-determinism of  $f$
- exclude non-determinism originating from arguments
- order-independent encapsulation of non-determinism

```
coin = 0 ? 1       $\rightsquigarrow$  coins = {0, 1}
id x = x           $\rightsquigarrow$  idS v = {v}   for all values v

bigCoin = 2 ? 4
f x = coin + x
fS bigCoin  $\rightsquigarrow$  {2, 3} or {4, 5}
```



## *n*-queens puzzle

Place  $n$  queens on an  $n \times n$  board without capturing:

- represent placement by a permutation (row of each queen)
- choose a *safe* permutation

A permutation is not safe if some queens are in the same diagonal:

```
unsafe (_++[x]++y++[z]++) = abs (x-z) == length y + 1
```

```
queens n | isEmpty (unsafes p) = p  
  where p = permute [1..n]
```

Note: use of set function is important here  
(all occurrences of  $p$  must denote the *same* permutation!)



Application areas: areas of individual paradigms +

## Functional logic design patterns [FLOPS'02, WFLP'11]

- **constraint constructor**: generate only valid data (functions, constraints, programming with failure)
- **locally defined global identifier**: structures with unique references (functions, logic variables)
- ...

## High-level interfaces for application libraries

- GUIs
- (type-safe) web programming
- databases
- string parsing
- testing
- ...



## Graphical User Interfaces (GUIs)

- layout structure: hierarchical structure  $\rightsquigarrow$  algebraic data type
- logical structure: dependencies in structure  $\rightsquigarrow$  **logic variables**
- event handlers  $\rightsquigarrow$  **functions** associated to layout structures
- advantages: compositional, less error prone

## Specification of a counter GUI

```
Col [Entry [WRef val, Text "0", Background "yellow"],  
      Row [Button (updateValue incr val) [Text "Increment"],  
          Button (setValue val "0") [Text "Reset"],  
          Button {exitGUI [Text "Stop"]}]]  
where val free
```



## MCC (Münster Curry Compiler)

- compiles to C
- supports programmable search, real arithmetic constraints

## PAKCS (Portland Aachen Kiel Curry System)

- compiles to Prolog
- non-determinism by backtracking, various constraint solvers

## KiCS2 (Kiel Curry Compiler Vers. 2)

- compiles to Haskell (fastest for deterministic programs)
- various search strategies  
(depth-first, breadth-first, iterative deepening, parallel)
- programmable encapsulated (demand-driven) search

... (or try <http://www-ps.informatik.uni-kiel.de/smapi/>)



## Combining declarative paradigms is possible and useful

- functional notation: more than syntactic sugar
- exploit functions: better strategies without losing generality
- needed narrowing: sound, complete, optimal
- demand-driven search  $\rightsquigarrow$  search space reduction
- residuation  $\rightsquigarrow$  concurrency, clean connection to external functions
- more declarative style of programming: no cuts, no side effects, . . .
- appropriate abstractions for high-level software development

One paradigm: **Declarative Programming**